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Modelling Synthetic Unit Hydrograph For Small Watershed Using Gamma Distribution Function

Pallavi Vitthal Patil
Assistant Professor, Agricultural Engineering, College of Agriculture, Baramati Affiliated to Mahatma Phule Krishi Vidyapeeth, Rahuri

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Abstract

The unit hydrograph of a watershed can be convoluted with the effective rainfall generated in the watershed to obtain the direct runoff hydrograph. This direct runoff hydrograph can then be utilized for designing the hydraulic structures in the watershed. However, the unit hydrograph development needs stream flow data of the watershed. In the absence of such stream flow data, the obvious choice is synthetic unit hydrograph development. Different methods of synthetic unit hydrograph development have been tried by hydrologists and they have suggested modifications to be done in different methods.

This paper uses the SCS dimensionless unit hydrograph approach to obtain time to peak from duration of effective rainfall and time of concentration of watershed. Moreover, the volume of unit hydrograph on rising and recession sides was considered along with the gamma distribution function to model the entire hydrograph.

The model was applied to two watersheds situated in different climatic conditions and having different areas. The model gave satisfactory estimates of time to peak, time base and peak discharge in both the watersheds. With the better model the absolute relative error in time to peak, time base and peak discharge for Aagadgaon watershed in Ahmednagar district (Maharashtra, India), was found to be 16.67, 30, and 7.17 per cent, respectively. In case of Shenda Park watershed in Kolhapur (Maharashtra, India) these were found to be 54.54, 10.71, and 6.38 per cent, respectively.

Keywords: Synthetic Unit hydrograph, SCS dimensionless unit hydrograph, small watershed, Gamma distribution function

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Introduction

The hydrograph can be regarded as an integral expression of the physiographic and climatic characteristics that govern the relationship between rainfall and runoff of a particular watershed. **Sherman** (1932) in his well known theory of unit hydrograph postulated that there is a linear relationship between effective rainfall and direct runoff. The unit hydrograph of a watershed when convoluted with effective rainfall generated in the watershed gives the direct runoff hydrograph, which can be the basis of design for hydraulic structures. However, derivation of unit hydrograph for a watershed needs stream gauging. In most of the cases the stream gauging data are not available, which poses the problem in deriving unit hydrograph for the watershed.

Snyder (1938) proposed answer to this difficulty with his synthetic unit hydrograph concept, wherein the Snyder's constants, C_t and C_p, are used for computing time to peak and peak discharge. **Gaddi and Cowen** (1999) modeled dimensionless synthetic unit hydrograph using four different regions of hydrograph. They considered that dimensionless unit hydrograph ordinate can be expressed as function of dimensionless time in different regions of hydrograph thus avoiding the problem of time step of rainfall and runoff hydrograph in the data set. **Das** (2000) have reported that the values of C_t and C_p vary over wide range (0.3 to 6.0) in different studies carried out world over. **Atre *et al*.** (2005) tried to model synthetic unit hydrograph for small watershed (0.1866 sq km) using Snyder method, SCS dimensionless unit hydrograph, and SCS triangular hydrograph. They concluded that all the methods need modifications in order to use them for development of synthetic unit hydrograph but SCS dimensionless unit hydrograph method gives better estimates than other two. **Jena and Tiwari** (2006) modeled unit hydrograph parameters, viz. time to peak, time base and peak discharge, using geomorphologic parameters of the watershed. They showed that hydrograph parameters can be modeled using pertinent geomorphologic parameters by using non linear regression equations.They also proposed that synthetic unit hydrograph is better alternative than Geomorphologic Instantaneous Unit Hydrograph (GIUH) in ungauged watershed.

Mane and Atre (2007) tried to compute t_p and Q_p for Aagadgaon watershed in Ahmednagar district (Maharashtra, India), with the constants, C_t and C_p, proposed by Snyder and found

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that the results for t_p and Q_p are not matching with observed values. They also determined the values of C_t and C_p from observed hydrographs. These were in the range given by **Das** (2000) by\ut not matching the values proposed by **Snyder** (1938). Thus the constants proposed by Snyder need modifications in different climatic and geomorphologic situations.

The shape of unit hydrograph is some what skewed and the area under the hydrograph is equal to unit volume of effective rainfall and hence gamma distribution function has attracted the attention of many hydrologists for modeling unit hydrograph. Rana (2001) tried to model unit hydrograph using different distribution functions, viz. Normal, Log-Normal, Gamma, etc. He concluded that for short duration unit hydrograph, Gamma function modeled the unit hydrograph satisfactorily. Patil (2004) attempted to model unit hydrograph for small experimental watershed at Shenda Park, Kolhapur (Maharashtra, India) using Gamma distribution function.

In this paper an attempt is made to model synthetic unit hydrograph with Gamma distribution function using SCS dimensionless unit hydrograph assumptions.

Model Development

The Gamma distribution function can be written as,

$$f(x) = \frac{\beta^{\alpha} x^{(\alpha-1)} e^{-\beta x}}{\Gamma(\propto)} \quad ...(1)$$

By putting time as independent variable and unit hydrograph ordinate instead of *f(x)* with suitable multiplying factor (Edson, 1951), the equation can be written as:

$$U(T,t) = \frac{2.78 \times A \times \beta^{\alpha} t^{(\alpha-1)} e^{-\beta t}}{\Gamma(\propto)} \quad ...(2)$$

By differentiating the equation with respect to ‘t’ and equating to zero to obtain maximum unit hydrograph ordinate.

$$0 = \frac{2.78 \times A \times \beta^{\alpha} \times t^{(\propto-1)} \times e^{-\beta t}}{\Gamma(\propto)} \times t^{(\alpha-1)} \left[-\beta + (\alpha-1)\frac{1}{t}\right] \quad ...(3)$$

Thus,

$$\beta = \left(\frac{\alpha-1}{t}\right) \quad ...(4)$$

This situation will occur for $t=t_p$. Substituting the value of β from equation (4) and replacing *U(T,t)* by peak discharge Q_p and t by t_p, in equation (2). The simplification will yield following equation.

$$\frac{Q_p \times t_p}{2.78 \times A} = \frac{(\propto -1)^{\alpha} e^{-(\alpha-1)}}{\Gamma(\propto)} \quad \ldots(5)$$

If observed unit hydrograph is available for a watershed then by putting values of Qp and tp, the shape and scale parameters of Gamma distribution function can be evaluated. But in absence of these data one has to depend on synthetic unit hydrograph assumptions.

Using the synthetic unit hydrograph assumptions for solving equation (5), considering the SCS dimensionless unit hydrograph method (**Singh**, 1994) we can obtain following relation.

$$Q_p = \frac{0.375 \times 2 \times Volume}{t_p} \quad \ldots(6)$$

This equation can be simplified as

$$Q_p = \frac{0.75 \times 2.78 \times A}{t_p} \quad \ldots(7)$$

Where, A is area of watershed in sq. km and t_p is time to peak in h.

Now substituting value of Q_p from equation (7) in equation (5) we get the relation as

$$0.75 = \frac{(\propto -1)^{\alpha} e^{-(\alpha-1)}}{\Gamma(\propto)} \quad \ldots(8)$$

The peak discharge of unit hydrograph can also be obtained from consideration of SCS dimensionless unit hydrograph as

$$Q_p = \frac{0.625 \times 2 \times Volume}{t_b - t_p} = \frac{1.25 \times Volume}{t_b - t_p} \quad \ldots(9)$$

Mutreja (1986) suggested that t_b is about 3 to 5 times t_p for small wastersheds. Considering very small watershed the value of time base, t_b, can be considered as $3t_p$. Therefore, we get following relation:

$$Q_p = \frac{1.25 \times Volume}{(3-1) \times t_p} = \frac{0.625 \times 2.78 \times A}{t_p} \quad \ldots(10)$$

Now substituting the value of Q_p from equation (10) in equation (5) we get the constant value of $\frac{Q_p \times t_p}{2.78 \times A}$ as 0.625.

Using the values of constants as 0.75 and 0.625 and equation (5), the valuess of shape parameter α, can be determined iteratively. The computation of t_p can be dome from SCS dimensionless unit hydrograph considerations.

$$t_p = \frac{D}{2} + t_L \quad \ldots(11)$$

$$t_L = 0.6 \times t_c \quad \ldots(12)$$

The time of concentration, t_c, can be determined using **Kirpich** (1940) equation.

$$t_c = 0.00032 \times L^{0.77} S^{-0.385} \quad \ldots(13)$$

Where,

t_c is time of concentration in h, L is length of main channel in the watershed in m and S is slope of main stream channel in m/m.

The scale parameter, β, can be computed from equation (4) using α and t_p. Thus by using equation (2) the unit hydrograph can be developed which is without any stream flow measurement and hence a synthetic unit hydrograph having duration equal to D-h.

Application of the Model

In order to apply the model, two watersheds, viz. Aagadgaon, Dist. Ahmednagar (Maharashtra, India) and Shenda Park, Kolhapur (Maharashtra, India) were selected. The rainfall as well as runoff data were monitored and average unit hydrographs for the watersheds were developed. The pertinent watershed characteristics and unit hydrograph parameters for Aagadgaon watershed (Mane, 1995) and Shenda Park watershed (Patil, 2004) are given in Table 1.

Table 1. Important watershed characteristics and average unit hydrograph parameters.

Characteristics	Aagadgaon Watershed	Shenda Park Watershed
Area, sq. km	1.73	0.12
Time of concentration, h	0.4833	0.20
Duration, h	0.50	0.25
Time to peak of average unit hydrograph, h	0.66	0.55
Time base of average unit hydrograph, h	3.0	1.40
Peak discharge, cum/s	6.0	0.80

The shape and scale parameters for the Aagadgaon watershed with $\frac{Q_p \times t_p}{2.78 \times A}$ constant as 0.75 were found to be 4.70, and 6.852, respectively. However, with constant as 0.625 these were 3.62, and 4.852, respectively. In case of Shenda Park watershed these parameters were found to be 4.70, and 15.102 for constant as 0.75 and 3.62, and 10.694 for constant as 0.625.

Thus the equations for determination of unit hydrograph ordinates for Aagadgaon watershed were obtained as

$$U(T,t) = \frac{2.78 \times A \times 6.852^{4.7} t^{(4.7-1)} e^{-6.852t}}{\Gamma(4.7)} \quad \ldots(14)$$

And

$$U(T,t) = \frac{2.78 \times A \times 4.852^{3.62} t^{(3.62-1)} e^{-4.852t}}{\Gamma(3.62)} \quad \ldots(15)$$

The equations for Shenda Park watershed would be

$$U(T,t) = \frac{2.78 \times A \times 15.102^{4.7} t^{(4.7-1)} e^{-15.102t}}{\Gamma(4.7)} \quad \ldots(16)$$

And

$$U(T,t) = \frac{2.78 \times A \times 10.694^{3.62} t^{(3.62-1)} e^{-10.694t}}{\Gamma(3.62)} \quad \ldots(17)$$

The synthetic unit hydrograph obtained with equation (14) to (17) have the duration equal to D, considered in computation of t_p. These unit hydrographs can be compared with the average unit hydrograph of the two watersheds for the three important parameters; time to peak, time base, and peak discharge.

Results and Discussion

The unit hydrograph ordinates of magnitude less than $1X10^{-4}$cum/ s were considered as zero. The computed synthetic unit hydrograph using α=4.7, approach 'A' and α=3.62, approach 'B' are depicted with average unit hydrograph of Aagadgaon watershed in Fig. 1. Similarly, the synthetic unit hydrographs and average unit hydrograph of Shenda Park watershed are compared in Fig. 2. It can be revealed from these figures that both the synthetic unit hydrographs by approach 'A' and approach 'B' have the peak discharge earlier than the average unit hydrograph. The time base is also more than average unit hydrograph except in approach 'A' for Shenda Park watershed. The peak discharge computed by approach 'B' is more close to peak discharge of average unit hydrograph for Aagadgaon watershed and Shenda Park watershed. The time to peak, time base and peak discharge for synthetic unit hydrographs and average unit hydrographs for Aagadgaon watershed and Shenda Park watershed are given in Table 2 along with the relative error.

Table 2. Comparision of unit hydrograph parameters for Aagadgaon and Shenda park watersheds.

Parameter	Unit hydrograph for Aagadgaon watershed			Unit hydrograph for Shenda Park watershed		
	Modeled by Approach 'A'	Modeled by Approach 'B'	Average	Modeled by Approach 'A'	Modeled by Approach 'B'	Average
Peak discharge, cum/s	6.679 (-11.32)	5.570 (7.17)	6.000	1.021 (-27.63)	0.851 (-6.38)	0.800
Time to peak, h	0.55 (16.67)	0.55 (16.67)	0.66	0.25 (54.54)	0.25 (54.54)	0.55
Time base, h	3.15 (-5.00)	3.90 (-30.00)	3.00	1.30 (7.14)	1.55 (-10.71)	1.40

(Figures in parenthesis show relative error with respect to average unit hydrograph parameters in per cent)

The relative error of different unit hydrograph parameters of synthetic unit hydrographs developed by approach 'A' and approach 'B' from average unit hydrograph ranged from 16.67 to 54.54 per cent for time to peak for Aagadgaon watershed and Shenda Park watershed, respectively. The range of relative error was (-) 30.00 to 7.14 per cent for time base of UH for Aagadgaon watershed and Shenda Park watershed, respectively. The relative error in case of peak discharge ranged from (-) 27.63 to 7.17 per cent for Shenda Park watershed and Aagadgaon watershed, respectively.

The peak discharge was over estimated by both the approaches for Shenda Park watershed. However, for Aagadgaon watershed approach 'A' overestimated and approach 'B' underestimated the peak discharge. The absolute deviation being less for approach 'B' in both watersheds.

The relative error was same for both the approaches in case of time to peak as the computation of time to peak for both the approaches was same. There was underestimation of time to peak for both the watersheds. However, the error was greater in case of Shenda Park watershed, which might be due to smaller size of the watershed.

The time base was over estimated by both the approaches for Aagadgaon watershed. However, for Shenda Park watershed approach 'A' underestimated and approach 'B' overestimated the time base. The absolute deviation being less for approach 'A' in both watersheds.

Considering all the three parameters; Q_p, t_p, t_b; of synthetic unit hydrograph the results of approach 'A' are found to be better than approach 'B'.

Conclusion

The synthetic unit hydrograph of a watershed, which can be used in the absence of stream flow data, can be modelled successfully with gamma distribution function and SCS dimensionless unit hydrograph approach for determining time to peak and peak discharge. When the methodology was applied to two different watersheds in altogether different climatic conditionsand with different areas, the results were surprisingly good for three important parametrs of unit hydrograph, viz. Q_p, t_p, and t_b. The approach 'A' modelled the synthetic unit hydrograph near to average unit hydrograph of the given watersheds.